School of Physics

Senior Honours Project
Physics

The motion of asteroid 3753 Cruithne – Earth's second Moon?

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Abstract

The peculiar path of near Earth asteroid 3753 Cruithne was revealed in a paper by Paul Wiegert, Kim Innanen and Seppo Mikkola published in Nature on June12, 1997. The aim of this project is to write a computer program based on initial conditions supplied by Paul Wiegert. The program should be able to simulate and visualize the orbit of Cruithne and allow to investigate the stability of the orbit.

Declaration:

I declare that this project and report are my own work.

Signature: Date:

Table of Contents

Abstract 1
Introduction 3
Theory 4
Cruithne's Motion explained: 4
Physical background: 6
Gravity explained: 6
Method 9
Obstacles encountered during coding: 10
Validating the code: 10
Results 15
Helio Centric frame: 15
Cruithne's closest approach to Earth: 17
Geo centric frame: Cruithne as seen form Earth: 19
Stroboscopic version of Helio centric frame 24
Co-rotational Earth frame: Revealing Cruithne's Horseshoe type orbit 26
Hitting Earth with Cruithne, a bit of Science Fiction 27
Discussion 29
The duration of one Earth year: 29
The orbital time of the horseshoe orbit in the co-rotational frame: 29
Conclusion 30
References 30
Appendix 31

Introduction

Asteroid 3753 was first discovered by J. Duncan Waldron on the 10th of October in 1986 and classified as a Near-Earth asteroid or Near-Earth-Object, simply called NEO. NEO's are objects which orbits bring them closer than 0.3 Au to Earth (1 Au or one Astronomical Unit is the distance between the Sun and Earth), and are sub classified in three types, depending on there orbits[1]. J. Duncan Waldron, as it's discoverer, also had the honour of naming asteroid 3753 and choose "Cruithne". Which is the name of the first Celtic racio-tribal group to come to the British Isles, sometime around 800-500BC, also known as Picts[2].

Although discovered in 1986 it was not until 1997 that Paul Wiegert, Kim Innanen and Seppo Mikkola revealed the complexity of Cruithnes orbit in a paper which was published in *Nature*, and which made Cruithne famous as 'Earth second moon'.

The main motivation of the project is to write a computer simulation which will confirm the results published by Wiegert *et.al.* and to explain, clarify and visualize it, such that it is easily comprehendible by people without astronomical background knowledge. It will also allow the user of the online simulation[3] to gain a understanding of the stability of Cruithnes orbit by manually applying small changes to it's momentum and by observing the effects on its orbit in run time. Further more we will try to make a rough estimation about the necessary energy to change Cruithne's orbit in such a way that it would lead to a collision with Earth.

To accomplish this tasks, two versions of software were written. A non graphics version which is used for detailed data output (as .txt files) and which was used for all experiments, and an java applet which, in conjunction with a small web page, should provide a more visual approach to the topic.

The simulation is based on Newtonian Physics (i.e. Relativistic Effects were not taken in account) and a Runga Kutta 4 algorithm (a numerical integration method, similar to Euler but more sophisticated, will be explained in more detail in the Theory section of this paper) and is written in the Java™ Programming Language[4].

Obtained results agree qualitatively with Wiegerts results, but show discrepancies regarding orbiting time of Cruithne in the co-rotational Earth frame, which might have to do with initial coordinates or the use of different units. Suggestions for further tests and an explanation for the discrepancy is offered in the Results & Discussion sections of this report.

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[1] For more information on NEO's visit the following websites:
http://neo.jpl.nasa.gov/
http://www.nearearthobjects.co.uk/

[2] For more information on the Picts visit http://www.scotshistoryonline.co.uk/

[3] The applet version can be found at: http://ph.ed.ac.uk/~p0125205/Solar/Solar.htm

[4] Suns java homepage: http://java.sun.com/

Theory

Cruithne's Motion explained:

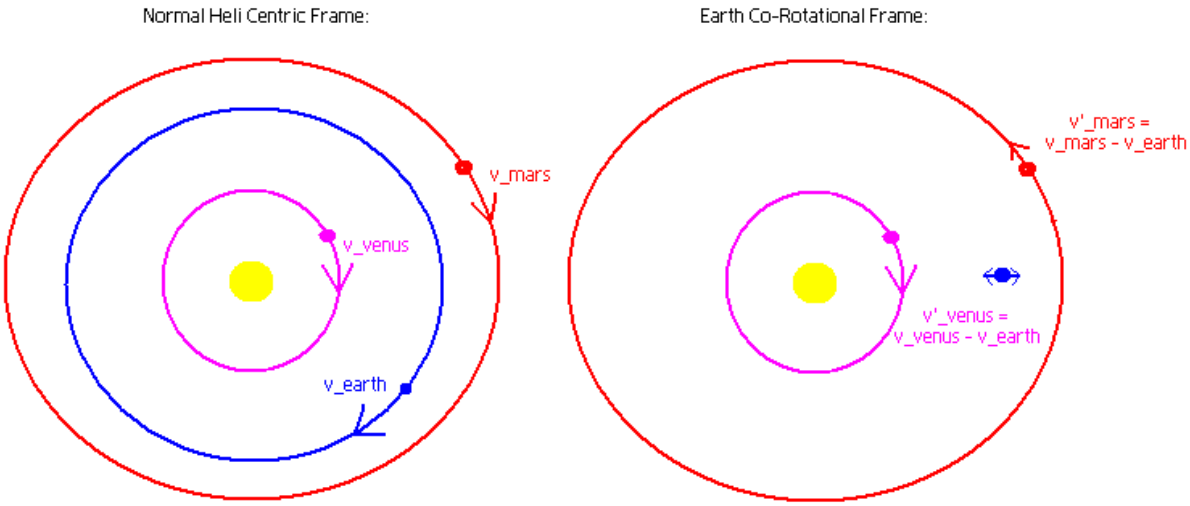
At first glance Cruithne appears to be an ordinary asteroid. Its only remarkable feature, compared to that of other planets, is the high inclination of almost 20°. It is this high inclination that reduces the chance of a colliding with another planet to almost zero and is also responsible for the rather late discovery of its peculiar orbital properties. A more detailed analysis reveals that Cruithne's orbiting time around the Sun is roughly the same as Earth's. This is what Astronomers refer to as a '1:1 Orbit Resonance'.

A 'Orbit Resonance' in astronomical terms means that the orbital periods of two bodies are related by factors of integers. Another example is the 2:3 resonance between Pluto and Neptune in our Solar System. This means that in the time Pluto takes to complete two full orbits, Neptune will complete three orbits[5].
The Resonance between Earth and Cruithne is not exactly 1:1 but is subject to slight changes over time. So Cruithne will sometimes be slightly faster than Earth and at other times slightly slower. This behaviour is not only due to the great influence of the Sun or the other planets, but also due to subtle gravitational interactions between Earth and Cruithne itself. To fully understand these influences one would need to solve a three dimensional restricted three-body problem of Cruithne, Earth and the Sun. 'Restricted' means that the mass of one of the bodies is much smaller than the other two and therefore neglected. Solving a three-body problem goes beyond the scope of this project but a paper[6] by Namouni et al. covers this topic.

By changing our view of the Solar system from the standard Helio-Centric frame to an 'Earth-co-rotational frame' the effects of the 1:1 Resonance become visible. A 'Earth-co-rotational frame is a frame in which the observer co rotates with Earth around the Sun. Imagine an observer is floating above Earths North pole and looking towards the sun. He or she would not realize that the Earth rotates around the Sun, but rather perceives the Sun to be stationary, with only small variations in the distance between Sun and Earth. These variations would be due to the fact that Earths orbit is not circular but slightly elliptical, thus bringing Earth sometimes closer or further away from the Sun. The planets closer to the Sun than Earth would still follow their orbits as usual except at slower velocities. The planets further away from the Sun than the Earth would appear to have changed direction, as the observer would only perceive the relative velocities of the planets with respect to Earth's own orbit velocity. Graph Nr.1 on the next page illustrates the difference between an Helio-Centric and an Co-rotational frame. The label *v_*planet name** indicate the velocity of the named planet in the Heli-Centric frame while *v´_*planet name** is the velocity in the Co-rotating frame.

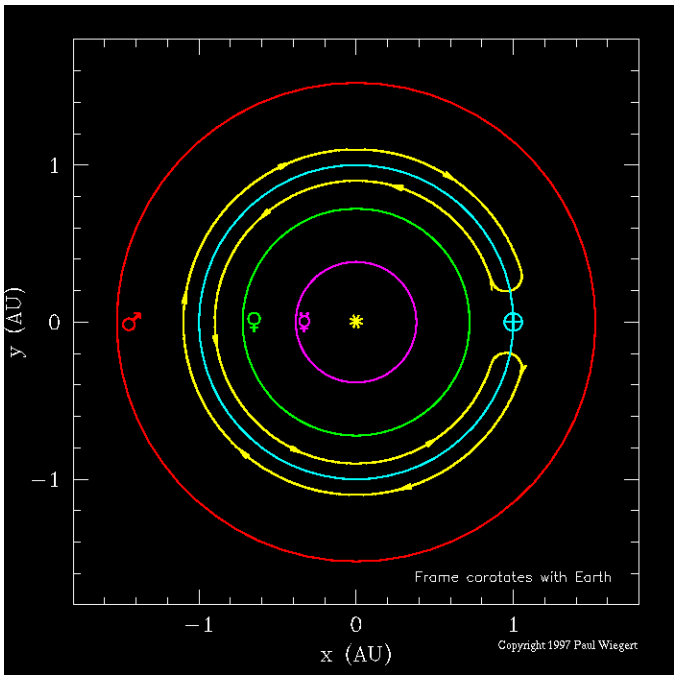
[5] For more information about Astronomical Resonance go
to: http://www.astro.uu.nl/~strous/AA/en//antwoorden/zwaartekracht.html - resonance
or to: http://math.arizona.edu/~goriely/M254-2004/resonances.pdf

[6] Namouni, 1999, Icarus, 137,293



Graph No.1

In this Co-rotational frame Cruithne reveals it's 'horseshoe' type orbit. Horseshoe type orbits draw their name from their unusual orbit form, which resembles a horseshoe. The *accompanying planet*[7] i.e. the planet it shares a orbital resonance with, is in the centre of the open section of the horseshoe. The following image (courtesy of Paul Wiegert) shows the theoretical shape (i.e. the yellow line) of such an horseshoe orbit:



Graph No.2

The orbit in the image is just a theoretical orbit. Cruithne's orbit is actually a lot more complicated. First of all it is not, as indicated above, an open horseshoe but an overlapping one, and secondly it does not trace out a simple circular orbit but one in which the circle is made of many very tightly wound spirals.
Cruithne's Orbit will be discussed further in the Result section of this report.

[7] A term embossed by Wiegert.

Physical background:

The physical principles which govern our Solar System are fairly simple compared to other branches of physics. The main motor driving it is Gravity and to a very small extent Relativity.
Relativistic effects only occur at high velocities, a good guess would be about 20% of the speed of light. But since the main bodies (i.e. the 9 planets) of our solar system are travelling a lot slower than that, one would not expect to observe any effects at all. However objects which come close to a huge mass such as the sun will experience relativistic effects as Space-Time around heavy masses is curved[8]. The best example for this is the orbit of Mercury. Observations showed that Mercury's orbit is processing with 572 arc seconds per century. Processing means that the elliptical orbit of mercury is not stationary but rotates around the Sun. An arc second is defined to be 1/360 degrees. Newtonian mechanics predicted this procession, caused by the pull of the other planets, but could only account for 539 arc seconds per century. It was not until Einstein applied his theory of general Relativity that the missing 33 arc seconds could be accounted for.
Since relativistic effects on Cruithne and Earth are negligible, they are not included in the simulation.

Gravity explained:

Since the times of the ancient Greeks, and maybe even earlier, humankind was intrigued by the movements of the bodies in the sky and thirsted for more knowledge and possible explanations. Most of them involved some gods driving or throwing things around the firmament and all had the Earth as the centre of the know Universe. It was not till the 16th Century when the work of Copernicus[9] and Kepler[10] placed the Sun at the centre of the Solar System and rekindled the scientific interest in it. In 17th Century Newton finally came up with the universal law of gravitation which he stated as follows:

Every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of this force is along the line joining the particles.

Expressing this statement in Vector form gives the following equations:

$$\underline{F_{21}} = -G\frac{m_1 m_2}{r_{12}^3}\underline{r_{12}} \qquad \text{(Eq.1)}$$

Where F_{21} is the force exerted on object two with mass m_2 by object one with mass m_1 and r_{12} is the displacement vector pointing from particle one to particle two. The minus sign in the equation indicates that the force points in the opposite direction than r_{12}, which means that it

[8] For more information about Space-Time visit: http://theory.uwinnipeg.ca/mod_tech/node60.html
[9] Biography of Nicolaus Copernicus: http://www.phy.hr/~dpaar/fizicari/xcopern.html
[10] Information about Johannes Kepler: http://www.kepler.arc.nasa.gov/johannes.html

acts towards body one. In other words, it is attractive. G is the universal constant of gravity, currently found by experiments to be $(6.6742 \pm 0.0010)\text{x}10^{-11}$ m^3 kg^{-1} s^{-2}.

A similar equation can be written down for the force exerted on object one by object two simply by interchanging the sub indexes 1 and 2.

$$\underline{F_{12}} = -G\frac{m_2 m_1}{r_{21}^3}\underline{r_{21}} \quad \text{(Eq.2)}$$

Noting that $\underline{r_{21}} = -\underline{r_{12}}$ in the above equations confirms that $\underline{F_{12}} = -\underline{F_{21}}$ which means that the gravitational force acting on the two bodies forms a action-reaction pair.

So although the underlying physical principle is very simple, its application to the solar system is not. Complications arise if more than two bodies are interacting with each other. Even though the so called ´3-Body-Problem´ can be solved by now, our mathematic techniques are still not good enough to solve a ´N-Body-Problem´. To visualize the complexity of the problem consider the following.
If we use Newton's second law to write Force as Mass times Acceleration i.e. $\vec{F} = m\vec{a}$, and acceleration as the second time derivative of position i.e. $\underline{a} = \frac{d^2\underline{r}}{dt^2}$ then Equations 1 & 2 can be rewritten as:

$$\frac{d^2\underline{r}}{dt^2} = G\frac{m(\Delta r)}{(\Delta r)^3} \quad \text{(Eq.3)}$$

Now, the acceleration acting on the Earth due to the other planets in terms of Cartesian coordinates is:

$$\frac{d^2 x_{earth}}{dt^2} = G\times\left[\frac{M_{sun}(x_{sun} - x_{earth})}{(D_{earth-sun})^3} + \ldots\ldots + \frac{M_{pluto}(x_{pluto} - x_{earth})}{(D_{earth-pluto})^3}\right]$$

which can be written as

$$\frac{d^2 x_i}{dt^2} = G\times\sum_{j}^{all\ planets}\frac{M_j(x_j - x_i)}{(D_{ij})^3} \quad \text{(Eq.4)}$$

and identical equations can be written for y and z coordinates .
Together with the equations for the other planets we get a total of 27 (3 dimensions *9 planets) coupled second order equations describing the system. If other bodies like Asteroids or Comets are included the number of equations is even greater, making the system a `N-Body-Problem`. Since we are unable to solve the system analytically we have to look for a numerical approach.

The first thing to realize is that we are dealing with second order differential equations, which can be reduced to a set of first order differential equations. We can for example rewrite equation 4 from above as:

$$\frac{dx_i}{dt} = \alpha_i(x)$$

and

$$\frac{d\alpha_i}{dt} = G \times \sum_{j}^{all\ planets} \frac{M_j(x_j - x_i)}{(D_{ij})^3}$$

Since our problem does not rely on boundary conditions but rather on initial conditions, we can approximate a solution to the above equations by rewriting the $d\alpha$'s and dt's as finite steps $\Delta\alpha$ and Δt and multiply both sides of the equations with Δt to get the following equations:

$$\Delta x_i = \Delta\alpha_i(x) \times \Delta t$$

and

$$\Delta\alpha_i(x) = G \times \left(\sum_{j}^{all\ planets} \frac{M_j(x_j - x_i)}{(D_{ij})^3} \right) \times \Delta t$$

This gives us formulas for the change in position and velocity of the planets when we make one "time step" of size Δt. If we make the time steps sufficiently small than we should get a good approximated solution of the underlying differential equation.
Proceeding through the function with small time steps is on the heart of each numerical integration algorithm. In fact this process is identical to Euler's[11] Method, however a more sophisticated method of numerical integration is used for this project (see Method section).

So it seems that the underlying physical principle is really easy. But is it really that easy? How can it work over such huge distances as our Universe? Why is it not limited in its strength? Why is it always attractive and never repulsive? What is this mysterious exchange particle called Graviton?
Discussing these questions would go beyond this project and are only meant to highlight the fact that even though we think to master the concept of gravity, there is actually way more to it than we understand yet.

[11] More detailed explanation about Euler method:http://qucs.sourceforge.net/tech/node22.html

Method

For simplification and performance reasons the following assumptions and changes where made:

- The sun does not move.
- Earth and Moon are combined in one body and placed at the centre of mass of the Earth – Moon system.
- Pluto is not included, as it is to far away and it's mass is too small to play any significant part in Cruithne's orbit.
- Paul Wiegert's initial conditions where converted into SI Units. This was done to simplify the momentum change option in the simulation but raised some problems due to the fact that Wiegert's year was slightly longer than a standard tropical year (see Discussion section of this report).

Different versions of the simulation where written, a number of **numerical** ones with **file output** for individual experiments and a **graphical** version. The graphical version[12] (a JavaApplet™) was programmed first and is meant for use via the internet. Its purpose is not only to enable the user to visualize Cruithne and the solar system, but also to highlight how peculiar and sensitive Cruithne's orbit actually is. This is achieved by allowing the user to look at the solar system in different reference frames, to change Cruithne's momentum at any time and to see the effects of this perturbation in runtime on the screen.

At the heart on the simulation lies a Runga Kutta 4 algorithm (RK4). A Java class implementing this algorithm was written by Tony Kennedy[13] and was used in the 3rd year Computer Simulation course and is also used in this project.
The RK4 is a much more sophisticated numerical integration algorithm than the Euler algorithm explained earlier in the Theory section of this report. A very good description of the RK4 algorithm can be found in the book *Numerical Recipes.* A online version of this book for the C™ Programming language is available at Cornell University (see Appendix).

Tony Kennedy's code was then sub classed in a *SolarIntegrator* class which implements a function method. This function method loops through all the planets, except the sun which is held at a fixed position in the centre of the system, and effectively differentiates the current position and velocity values (x, y, z, v_x, v_y, v_z) to get velocity and acceleration (v_x, v_y, v_z, a_x, a_y, a_z). The calculations of the new acceleration values, due to the gravitational influence of the other planets, are the highest computational expenditure in the code.

(For further information on code see source code and the corresponding JavaDoc™ documentation in the Appendix of this report).

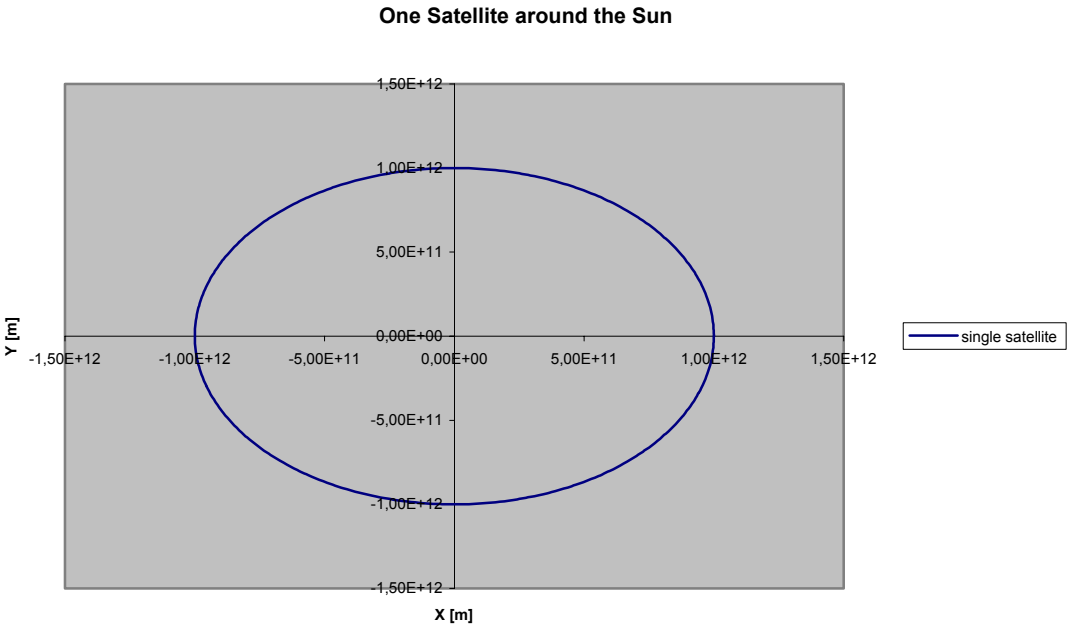
[12] Can be found here:http://ph.ed.ac.uk/~p0125205/Solar/StartMe.html
[13] Tony Kennedy is a lecturer at Edinburgh University School of Physics

Obstacles encountered during coding:

Running out of memory:
Since the applet allows the user to look at different reference frames and be able to draw the orbits in each frame, it will occupy more and more system memory with each calculated time step because it has to store all the orbits. Although the user has the option of manually clearing the orbit data in the active reference frame to get more free memory, there is still the danger that the applet uses up all the available memory and creates an 'out of memory error'. On an old PIII laptop with 256MB of RAM this happens after ~300 simulated years. The easiest approach to avoid this problem would be to clear all the orbit data after for example every 100 years. But since users will have different hardware, such an approach could still create an error on an old machine with little RAM or restrict the functionality of the applet on a new machine which might be able to store more than 1000 years of orbit data.
The solution to this dilemma was to write the code in such a way that it monitors the amount of memory available to the Java Virtual Machine™ running on the user machine and to clear the orbit data for all reference frames when it drops below 100kB of free memory.
An interesting aside is the fact that clearing the data alone was not good enough to free the memory, also a manual call to the garbage collection method of the JVM™ was necessary. This was unexpected as the JVM™ should handle garbage collection automatically.

Scalability of the solar system:
Another problem posed the dimension of the solar system itself. The applet should display all the planets, from Mercury out to Neptune with the distances correctly scaled. But Neptune is already so far away that if displayed on the edges of the screen the inner planets are hardly distinguishable because they appear to be clustered very tightly around the sun. The solution to this was to implement a 'magnification' factor that would allow the user to zoom closer into the area around the sun but to still keep the correct scaling factor.

Validating the code:
Once the coding was finished two analytic test where carried out to check if the simulation was working properly.
First a **single satellite** orbiting a **fixed** sun on a **circular** orbit (see Graph No.3):
Assumptions: Sun is fixed at (0,0,0); Sun's Mass $=1.989 \times 10^{30}$ kg; Satellites mass =1kg;
Orbiting radius $r = 10^{11}$ m



Graph No. 3

By equating the acceleration on circular orbits with the acceleration due to gravity one can calculate the orbit speed:

$$\frac{v_{orbit}^2}{r} = \frac{G \times M_{sun}}{r^2} \quad \Rightarrow \quad v_{orbit} = \sqrt{\frac{G \times M_{sun}}{r}}$$

Once the we know the orbit speed we can calculate the time it takes the satellite to complete it's orbit:

$$T_{orbit} = \frac{D_{orbit}}{v_{orbit}} = \frac{2 \times PI \times r}{v_{orbit}}$$

Substituting in the assumed values from above gives:

$$T_{orbit} = \sim 5.45 \times 10^8 \text{ sec.} \qquad v_{orbit} = \sim 11521.7 \, {}^{m}\!/_{\text{sec.}}$$

Using these results in the non-graphics simulation software and splitting the Orbit period in 365 days with ten RK4 steps to be carried out for each day gives the following result:

Start position (x, y, z, v_x, v_y, v_z) = (10^{11}m, 0.0, 0.0, 0.0, ~11521.7m/s, 0.0) ;

After the 365 calculated days the satellite should be back at the starting point. The calculated position is:

End position (x, y, z, v_x, v_y, v_z) = (10^{11}m, 1.29m, 0.0, -1.49x10^{-8}m/s, ~11521.7m/s, 0.0);

The 1.29m deviation from the expected result indicates an error in the range of 10^{-9} % compared to the orbit radius.

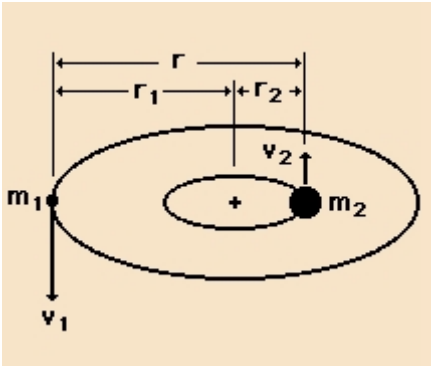
The test was repeated with the same initial condition but now with a 100 year run time, which means that 36500 days where calculated each with 10 RK4 steps each. This test was done to see if the error would increase linearly with the number of years calculated.

End position after 100 years = (10^{11}m, 295.3m, 0.0, -3.4x10^{-6}m/s, ~11521.7m/s, 0.0);

Even though the error did not scale up linearly from ~1.29m to roughly 129m, the 295.3m deviation are still a very good result considering the huge distances involved.
It would of course be possible to reduce this error by increasing the number of RK4 steps per day. For example using a hundred steps instead of ten would reduce the error from 1.29m to 0.006 m but then the performance would be too slow.
A last test with this set-up was done to see if changing the number of calculated steps would influence the outcome. The test run for 100 years was repeated but instead of calculating 365 days per year with 10 steps each, a whole year was calculated with 3650 steps each. The outcome was virtually identical with the one before. Instead of 295.3m the deviation was now 295.27m. So it was concluded that the number of integrations (i.e. either 365 per year or just a single one) per time interval play no significant part in the overall error.

The **second analytic test** contains now **two bodies** moving in **circular orbits around their centre of mass** (see Graph No.4 below).
Unfortunately this is the last analytic test possible, as going to three or more interacting bodies would require to solve a 3- or N-body problem.

Assumptions: Body-two has mass m_2=10^{25}kg; Body-one has mass m_1 = ½ m_2;
Distance from centre of mass for Body-two r_2=10^9m; r_1=2*r_2;



[14]

Graph No. 4

For the centre of mass to remain fixed, the two particles need the same angular velocity, which can be calculated by solving the orbit equation:

$$\frac{G \times m_1 \times m_2}{r^3} = \mu\omega^2 \qquad where \quad \mu = \frac{m_1 \times m_2}{m_1 + m_2}$$

[14] image from: http://hyperphysics.phy-astr.gsu.edu/hbase/orbv.html - co

Once the angular velocity is know, the orbital speeds and the orbital period can be calculated easily:

$$v_{orbit\ 1} = \omega \times r_1 \qquad \text{and} \qquad v_{orbit\ 2} = \omega \times r_2$$

$$T_{orbit} = \frac{2 \times PI \times r_1}{v_{orbit\ 1}} = \frac{2 \times PI \times r_2}{v_{orbit\ 2}}$$

Substituting the assumed values from above in this equations gives:

$$v_{orbit\ 1} =\sim 385.12\ \mathrm{m}/\mathrm{sec.} \qquad v_{orbit\ 2} =\sim 192.56\ \mathrm{m}/\mathrm{sec.}$$

$$T_{orbit} =\sim 3.263 \times 10^7\ \mathrm{sec.}$$

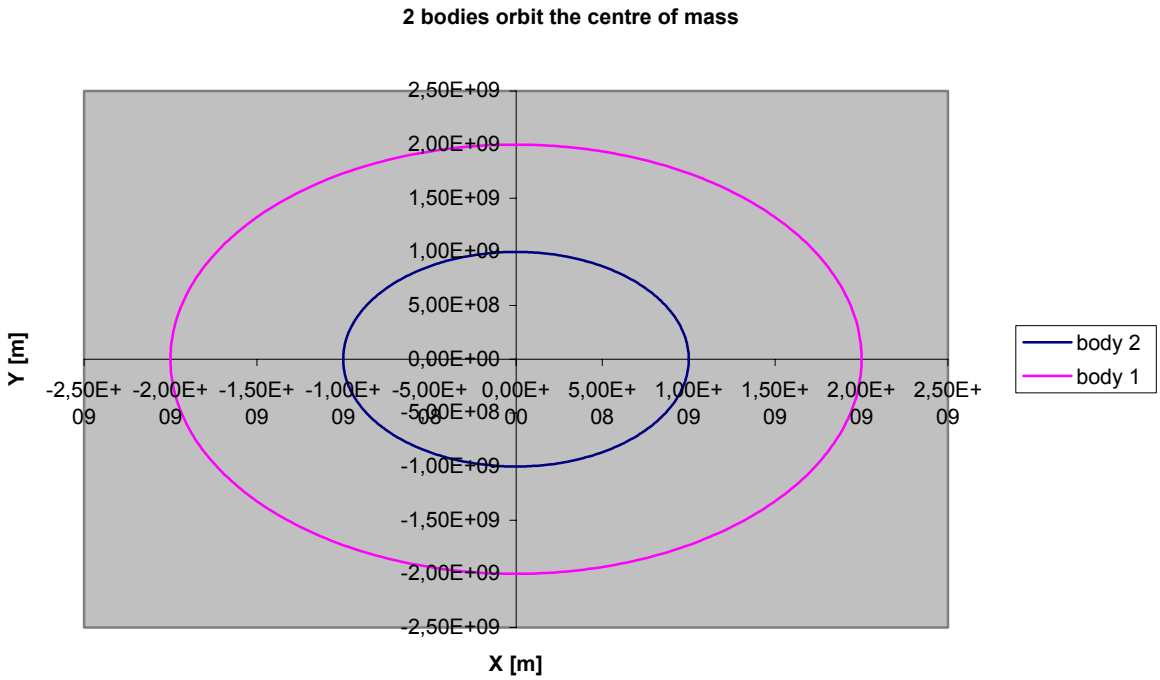
Using these values in the simulation software and splitting the Orbit period in 365 days with ten RK4 steps to be carried out for each day gave the following result:

Start position m_1 (x, y, z, v_x, v_y, v_z) = ($2x10^9$m, 0.0, 0.0, 0.0, ~385.12m/s, 0.0) ;
Start position m_2 (x, y, z, v_x, v_y, v_z) = ($-1x10^9$m, 0.0, 0.0, 0.0, ~-192.56m/s, 0.0) ;

After the 365 calculated days the satellite should have been back at the starting point. The found position was:

End position m_1(x, y, z, v_x, v_y, v_z) = ($2x10^9$m, 0.001m, 0.0, $-2.5x10^{10}$m/s, ~385.1m/s, 0.0);
End position m_2(x, y, z, v_x, v_y, v_z) = ($-1x10^9$m, -0.002m, 0.0, $5x10^{10}$m/s, ~-192.5m/s, 0.0);

The traced out orbits can be seen below. (Graph No. 5)



Graph No.5

Additionally to the x, y, z position the energy conservation was monitored. The total energy was calculated by adding the kinetic energy of each body and the potential energy of the system. The total energy of the system is of order $1x10^{29}$J, the fluctuation in the simulation were in the order of $1x10^{15}$J, which is **fourteen orders of magnitude smaller** than the total energy.

The observed errors in this two test were sufficiently small, so it was assumed that the simulation is working correctly.

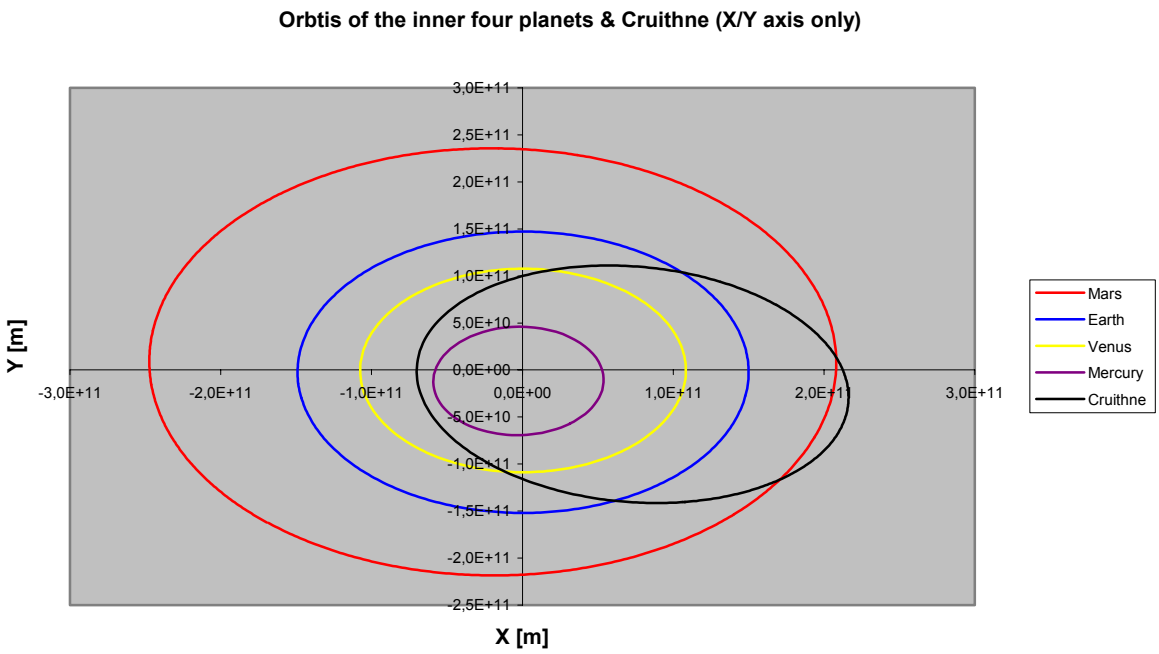
Results

Helio Centric frame:

- Setup: The simulation runs in steps of one Earth day (i.e. the position of all planets will be updated every day, which is ~86460 sec. in Wiegert's frame) with ten RK4 steps per day. All planet positions and velocities and the total energy of the system for each of the simulated days are written to a .txt file.

- Expected results: A small gain in the overall energy of the system over time due to rounding errors in the simulation. Obtained orbit periods should match the theoretical ones.

- Obtained results for the inner four planets and Cruithne: The simulated orbits for the inner four planets and Cruithne can be seen below. (Graph No. 6)



Graph No. 6

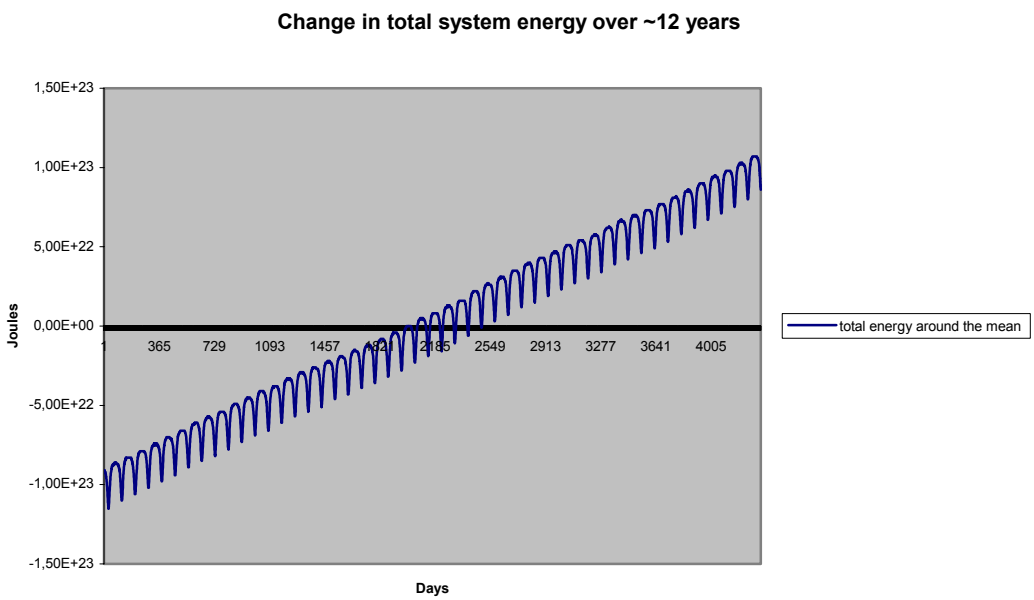
Comparing the obtained data with the theoretical orbit duration:

Planet:	Orbit duration:	Position:			
Mercury	~88 days		at day 1:	at day 87:	at day 88:
		X:	3,1242E+10	2,8744E+10	3,1637E+10
		Y:	-5,7175E+10	-5,9017E+10	-5,6861E+10
		Z:	-7,5381E+09	-7,4594E+09	-7,5487E+09
Venus	~244,7 days		at day 1:	at day 244:	at day 245:
		X:	-1,0516E+11	-1,0542E+11	-1,0478E+11
		Y:	-2,3247E+10	-2,2057E+10	-2,5025E+10
		Z:	5,7517E+09	5,7834E+09	5,7057E+09

Earth:	~365,24 days		at day 1:	at day 365:	at day 366:
		X:	-1,4658E+11	-1,4655E+11	-1,4605E+11
		Y:	-2,9339E+10	-2,9866E+10	-3,2394E+10
		Z:	1,9531E+05	2,5466E+05	2,6863E+05
Mars:	~687 days		at day 1:	at day 686:	at day 687:
		X:	-1,6849E+11	-1,6938E+11	-1,6784E+11
		Y:	-1,6259E+11	-1,6210E+11	-1,6342E+11
		Z:	7,3576E+08	7,6528E+08	6,9982E+08
Cruithne:	~ <= Earth		at day 1:	at day 364:	at day 365:
		X:	1,0737E+11	1,0604E+11	1,0807E+11
		Y:	-1,4037E+11	-1,4055E+11	-1,4031E+11
		Z:	-1,2046E+09	-7,9347E+08	-1,4332E+09

The results don't match with the expected ones exactly. However they are within a tolerance of less than +/- one day step which is the resolution of the simulation. Farther more, if one considers that the planets will not come back to the exact same position after one orbit period, due to precession[15] , then the obtained result is qualitatively good.

Analysing the total energy (i.e. kinetic energy of each planet and the total gravitational potential energy of the system) reveals a steady increase with small fluctuations. The small fluctuations have a period of 88 days which is also the orbital period of Mercury. This suggest that they are due to the strong elliptical orbit of Mercury, which brings the planet very close to the sun and therefore suggest that the change in kinetic/potential energy of the planet is larger than those of the other planets.
The graph below (Graph No.7) shows the increase of the total energy of the system around its mean energy. Even though this numbers are large, one has to keep in mind that the total energy is still a lot larger with a value of roughly $-2x10^{35}$ Joule, so although the increase is of order 10^{23} J, it is still **twelve orders of magnitude smaller** than the total energy.



Graph No. 7

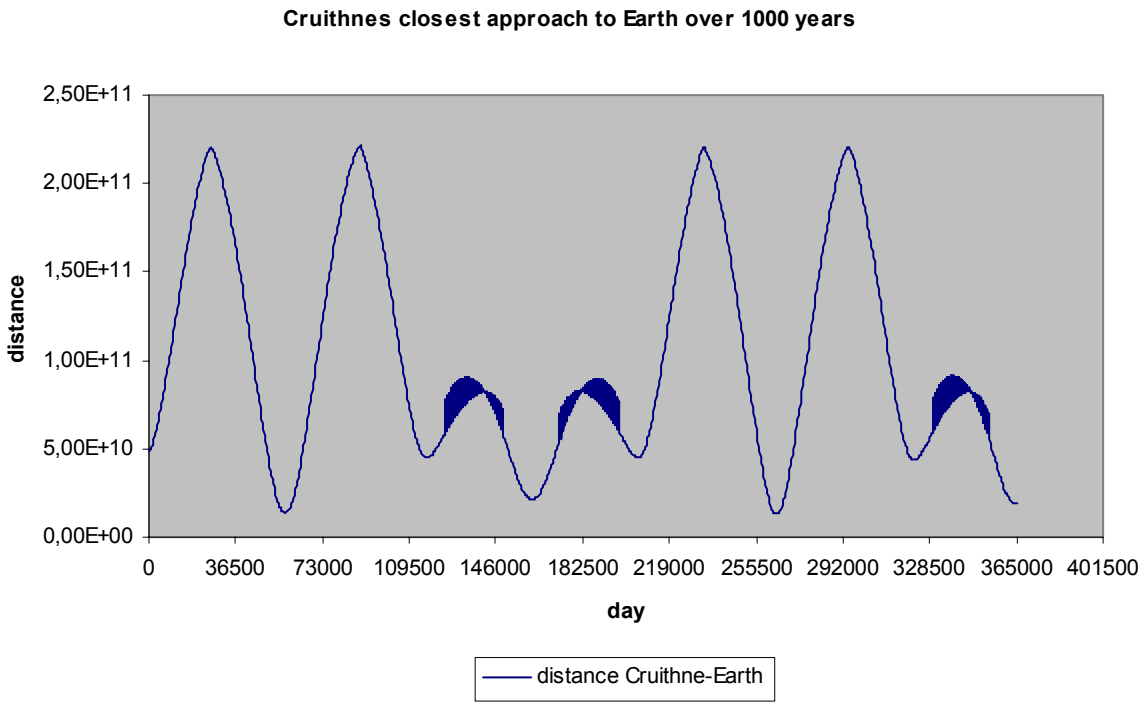
[15] Mercury as example for precession:
http://phyun5.ucr.edu/~wudka/Physics7/Notes_www/node98.html

Cruithne's closest approach to Earth:

- Setup: The simulation runs in steps of one Earth day (~86460 sec. in Wiegert's frame) with ten RK4 steps per day. Earth's and Cruithne's position and the closest approach value for each year is written to a .txt file. The closest approach distance per orbital cycle is calculated by comparing the distance between Earth and Cruithne for the actual day with the value from the day before. If the value of the actual day is smaller and stays smaller than the value from the day before, then Cruithne is approaching Earth. If the actual value is suddenly larger than the one form the day before, than Cruithne is moving away form Earth and the day before was the closest approach.

- Expected results: According to Wiegert, Cruithne should come as close as 0.1 Au or roughly 15 million kilometres every few hundred years.

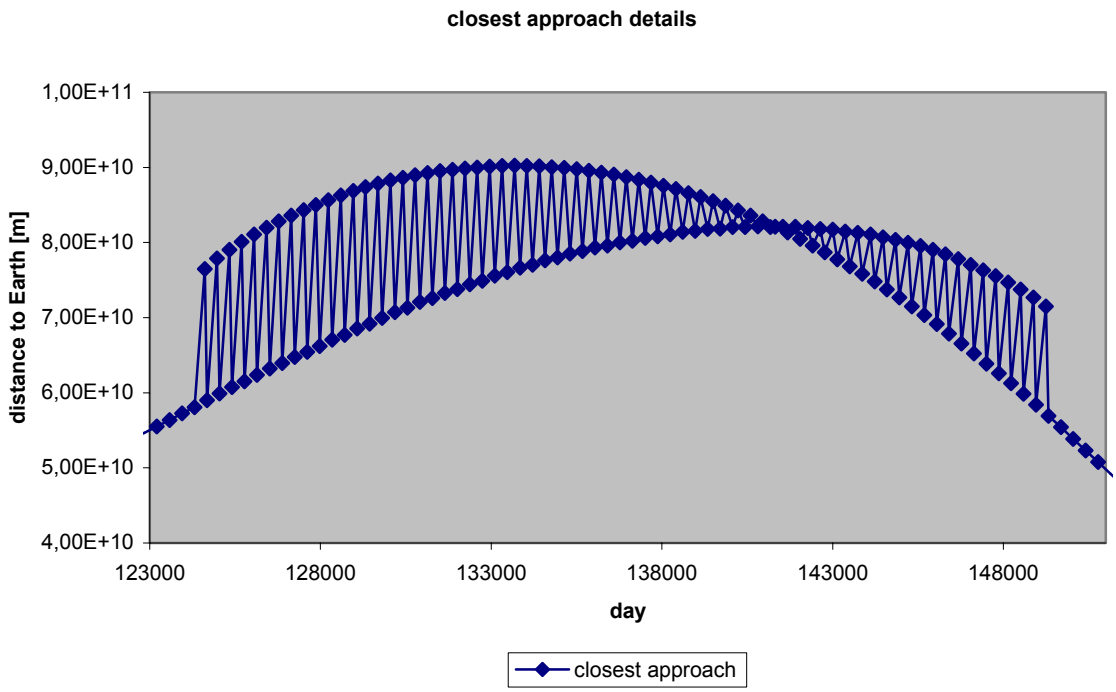
- Obtained results: Graph No. 8 shows the closest approach value per year over a duration of 1000 years.



Graph No. 8

The obtained result agrees qualitatively with what Wiegert says on his web page. The closest approach brings Cruithne closer than 0,1 Au to Earth. The simulated values for the first and second deep minimums in Graph No.8 above are:
14,08 million kilometres at day 57383 (~157,2 years) of the simulation and 13,22 million kilometres at day 263487 (~721,8 years) of the simulation. This suggest a period of ~565 years between successive closest approaches. But even more interesting is the region in between the two minima.

A closer look at the first dark area around day 140000 (Graph No. 9) reveals that there **are suddenly two closest approaches per year**.



Graph No. 9

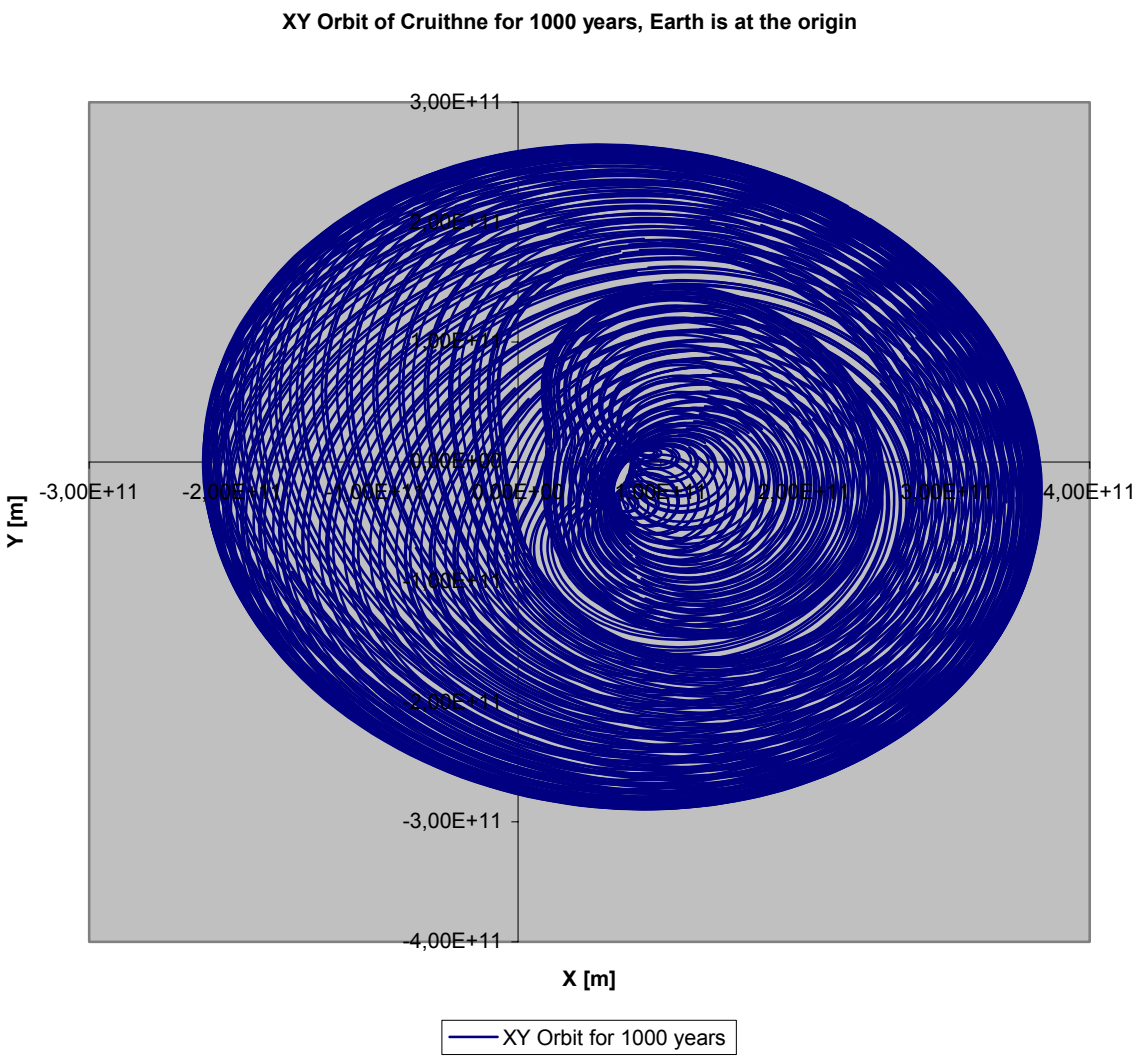
This has to do with the way the simulation calculates the closest approach distance. The little algorithm explained above was written with the assumption that Cruithne's "orbit" around Earth would always be elliptical. It seems that this is not always the case. The next experiment will try to explain this strange behaviour.

Geo centric frame: Cruithne as seen form Earth:

- Setup: The simulation runs in steps of 3.65 Earth days (~315580 sec. in Wiegert's frame) with 36 RK4 steps per simulation step. Cruithne's position relative to Earth will be calculated for 100 simulation steps (i.e. 365 days i.e. a year) every five years. This is done to minimize the amount of data acquired, because a simulation run of 1000 years would result in a .txt file with 365000 lines, which is way beyond the limit of what the most common spread sheet program can handle.

- Expected results: Unknown, since Wiegert does not include this reference frame in his report.

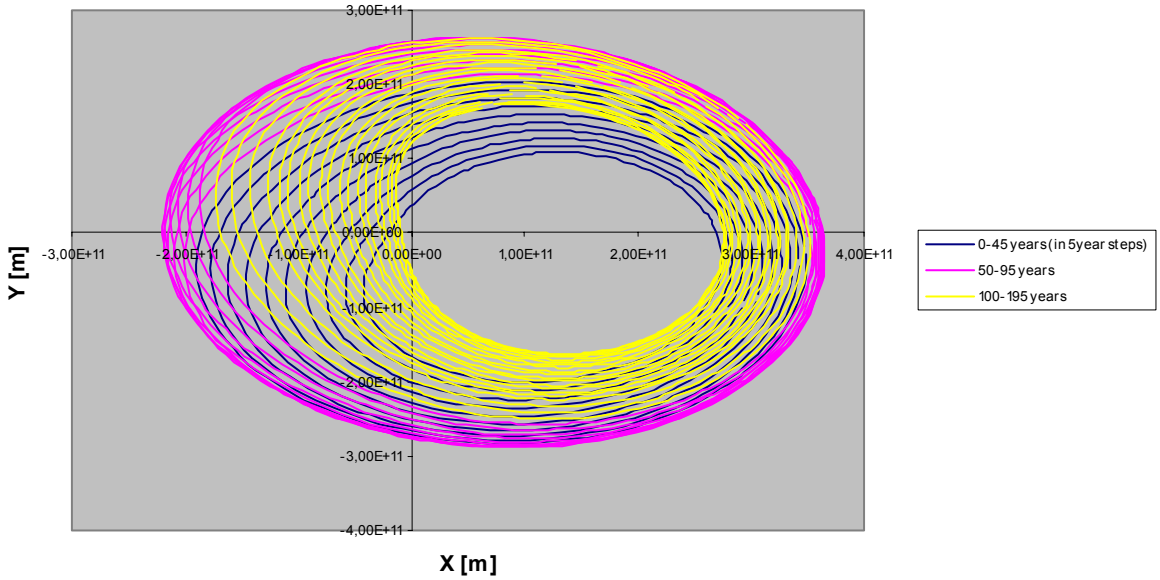
- Obtained results: The Geo centric view starts to reveal the uniqueness of Cruithne's strange relationship with Earth. Graph No. 10 shows Cruithne's orbit for 1000 years as seen from Earth.



Graph No. 10

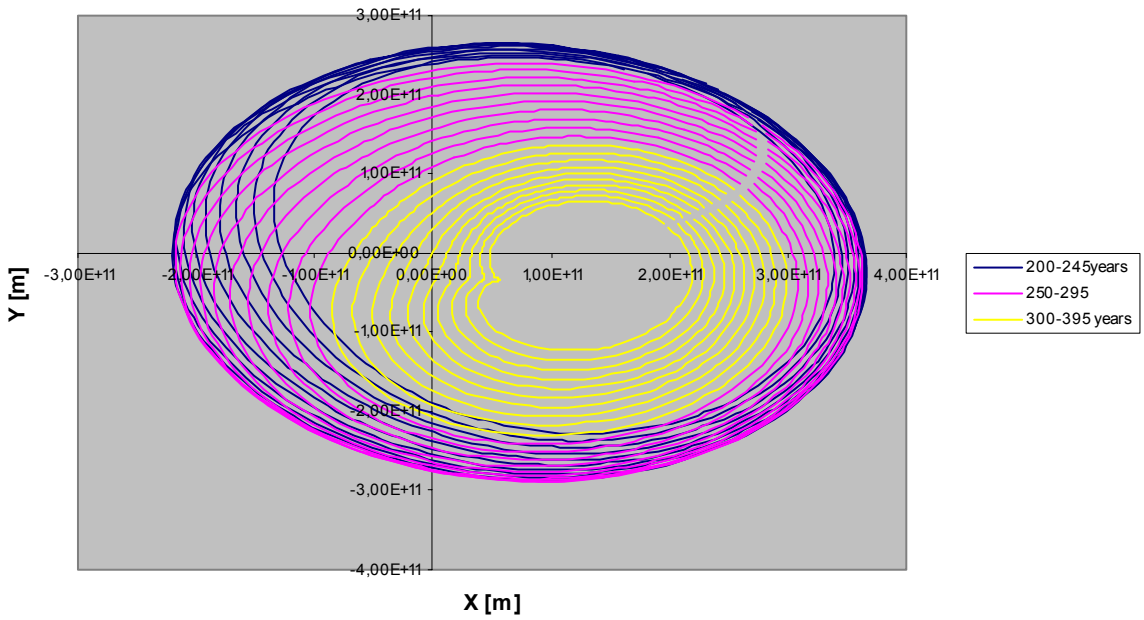
To interpret this rather strange orbit, it is necessary to analyse how it evolves with time.

By looking at the first 200 years, one can see that in the beginning (the simulation starts with the data from April 1st 2001) Cruithne does behave like a second moon. It appears to orbit Earth, but with an orbit that brings it farther and farther away form Earth (the blue line in Graph No. 11). In the time interval from 50 to 95 years (the pink lines) it reaches its maximum distance from Earth and appears to turn slightly and to start coming closer to Earth. Around 160 years after the simulation started it appears to slow down and to reverse its direction and to grow outwards again (yellow lines).



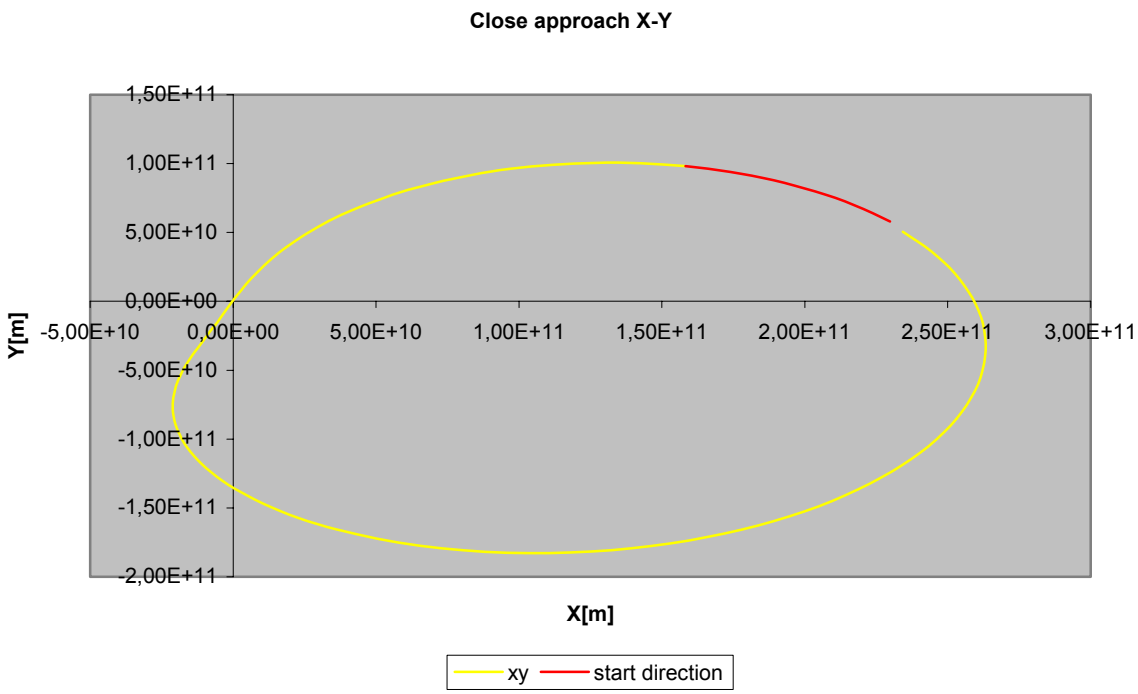
Graph No. 11

It continues to grow outwards until it reaches the maximum distance from Earth again(Blue lines in Graph No. 12). Then it reverses the turn it did when it reached the maximum distance for the first time, and comes closer to Earth again (pink and yellow lines).

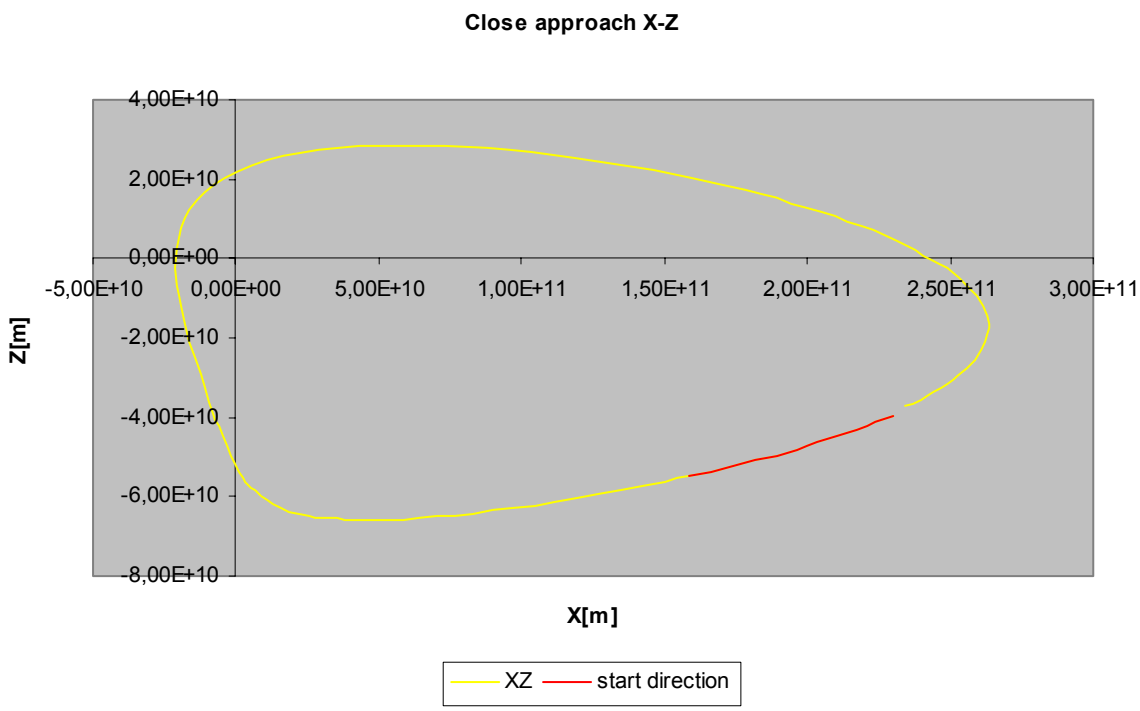


Graph No. 12

Except that it continues to shrink and it seems that it goes straight through Earth. A closer look at this particular orbit line reveals that Cruithne indeed crosses Earth's position almost exactly, but luckily passes by underneath the Earth. In the graphs No. 13 and 14, start at the gap and follow the red line to find the orbit direction.



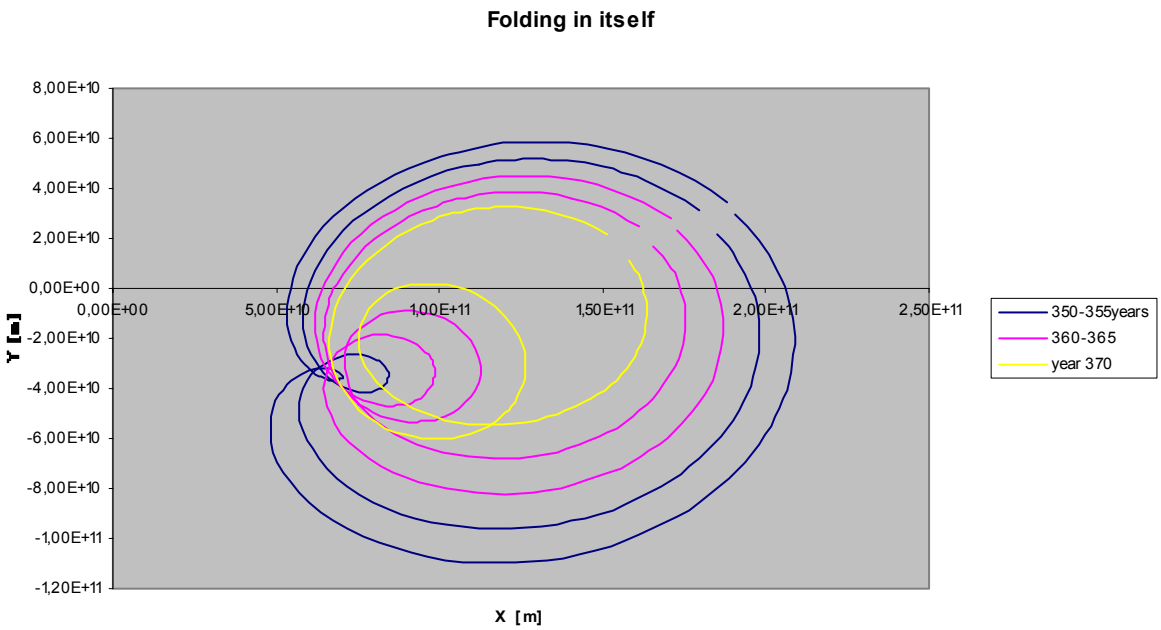
Graph No. 13



Graph No. 14

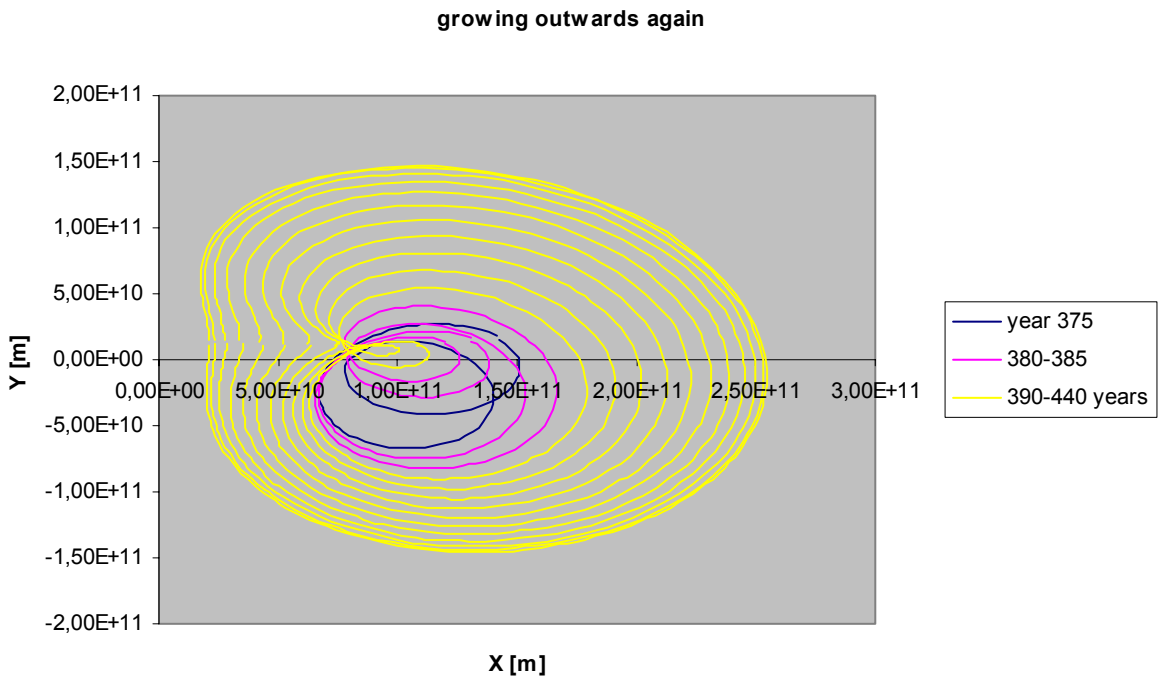
So when Cruithne passes through Earth's x/y-position, it is roughly 50 million kilometres underneath the Earths south pole. But even more interesting than that is the fact that when Cruithne goes past Earths position its orbit starts changing in a strange way.

It appears to be no longer circular but starts to form a kidney bean like orbit (yellow lines in Graph No. 12), that becomes more and more prominent the smaller Cruithne's orbit becomes. It is this kidney shaped orbit that is responsible for the "double closest approaches" from the closest approach experiment (Graphs No. 8 & 9).
Cruithne's orbit continues to become smaller while at the same time it seems to fold itself inwards (See Graph No. 15). The little in cut in the kidney bean orbit becomes sharper until it forms a little loop. This inner loop starts growing while the outer loop, the remains of the bean shape, becomes smaller until the inner loop is actually larger then the outer loop.



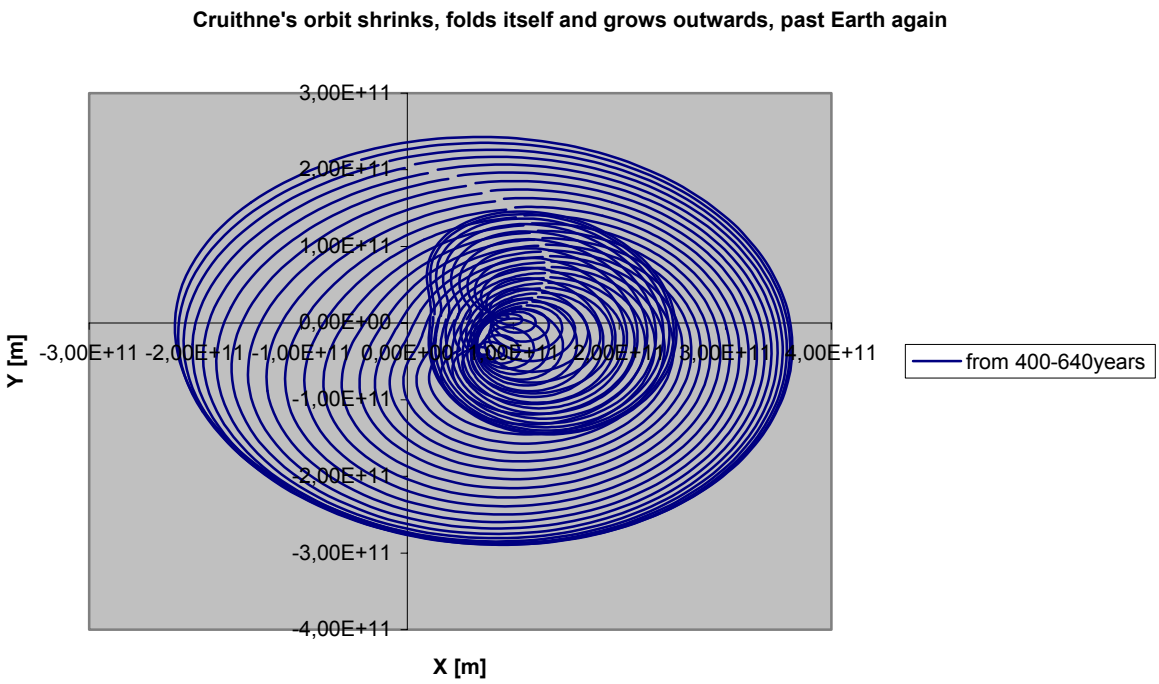
Graph No. 15

Once the folding is completed (blue and pink lines in Graph No. 16) it starts to grow outwards again (yellow lines), coming closer to Earth:



Graph No. 16

But before it reaches Earth, it slows down again, turns slightly, shrinks and folds again and then continues to grow outwards past Earth (Graph No. 17). It reaches the same orbit it had in the beginning of the simulation roughly ~570 years after the start of the simulation.



Graph No. 17

Combining all these results gives the very complicated orbital pattern shown on Graph No. 10, with a period of roughly ~570 years. The fascinating point about this highly complicated orbit is the fact that it only occurs due to the small changes in the orbit resonance between Earth and Cruithne. By observing either Earth or Cruithne independently, which are both in an stable elliptical orbit around the sun, one would never guess the complicated relationship those two bodies share.

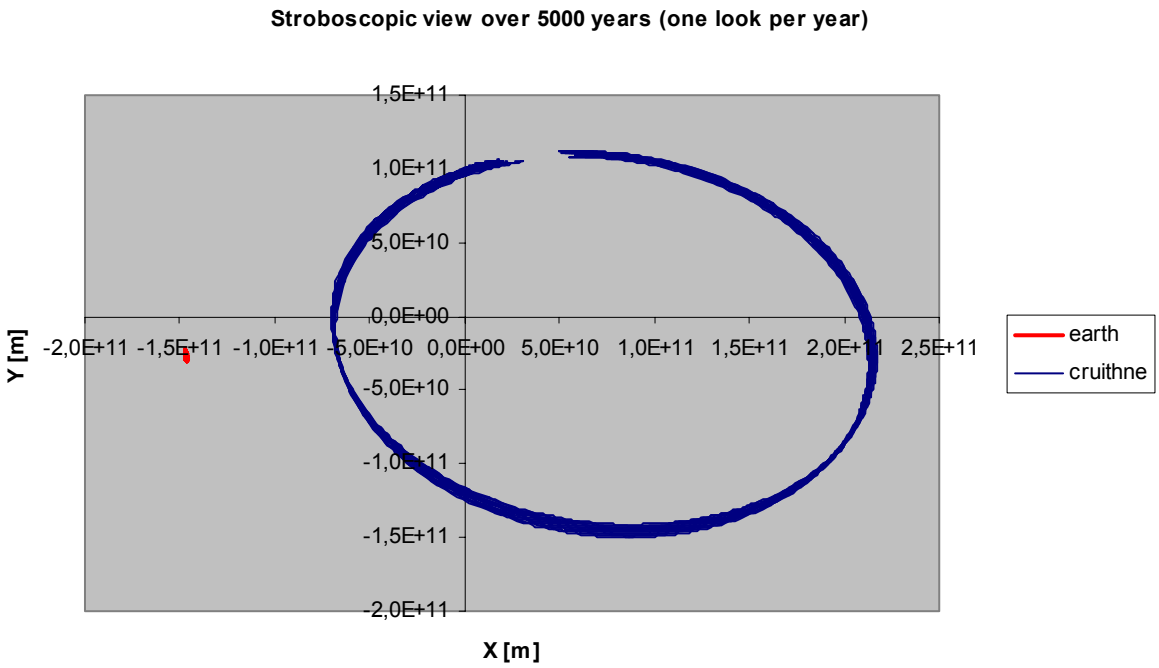
The next experiment will visualize how Cruithne's and Earth's orbit velocities are related.

Stroboscopic version of Helio centric frame

- Setup: The simulation runs in steps of a full year(~31558149 sec. in Wiegert's frame) with 3650 RK4 steps per simulation step. Cruithne's and Earths position and the total energy of the system are written to a .txt file once per simulation year.

- Expected results: Earth should stay at almost the same position, while Cruithne should sweep out a large part of its orbit. It should slowly move through it's orbit as it will be faster than Earth in the beginning, then slow down to the same orbital velocity as Earth, such that it will appear to be stationary in the stroboscopic frame. After reaching this stationary point it will slow down even more and start to move back along its orbit, as its orbital velocity is now slower than Earth's. After some time it will increase its orbital velocity again and the whole cycle will be repeated.

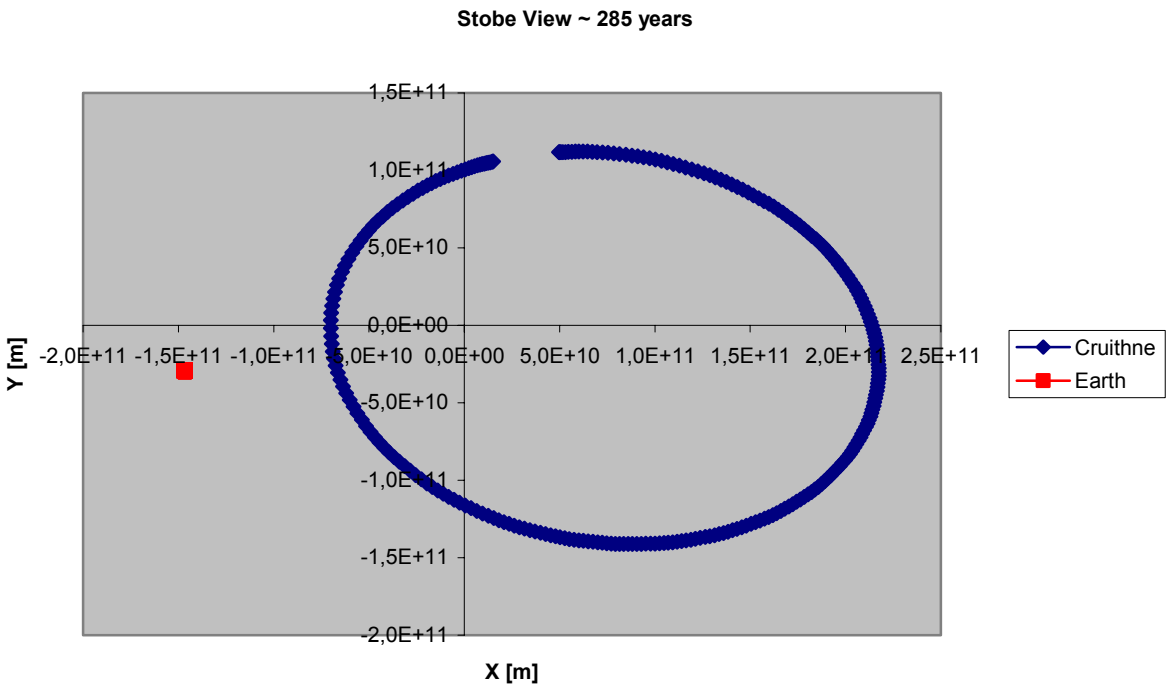
- Obtained result: After running the simulation for the first time it was found that Earth was not stable but slowly drifted along its orbit line. This means that the year Wiegert used in his simulation and the year in this simulation where slightly different. By adjusting the value of the year such that Earth would come back to roughly the same position each year, it was found that a day in the simulation is about ~86420 sec. while Wiegert's day is ~86460 sec. This sums up to a difference of roughly 4 hours per year (More information about this problem can be found in the discussion section of this report).
  The new values where used in the next experiment which gave the following result. Graph No. 18 shows how Cruithne sweeps out its orbit over 5000 years, going back and forth while Earth remains stable.



Graph No. 18

The time it takes Cruithne to complete one sweep, i.e. to go from one side of the opening to the other and back is roughly 570 years and fits to the periodic time discovered in the Geo centric frame experiment.

A single sweep, i.e. going from one end of opening to the other, takes ~285 years (Graph No.19).



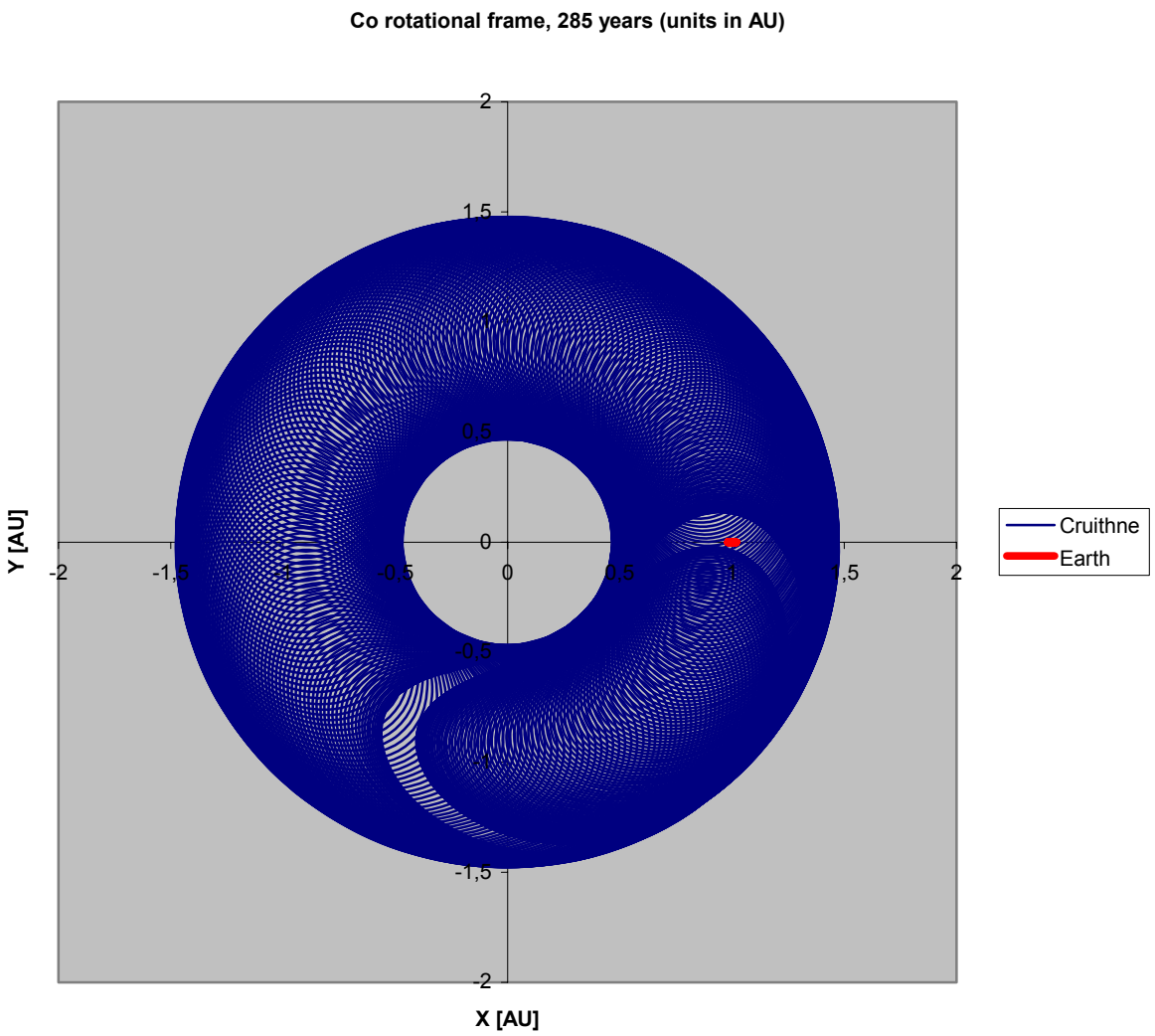
Graph No. 19

Co-rotational Earth frame: Revealing Cruithne's Horseshoe type orbit

- Setup: The simulation runs in steps of 0.04 Earth years with 146 RK4 steps per simulation step. Cruithne's position relative to Earth in a frame which co- rotates with Earth will be written to a .txt file.

- Expected result: Cruithnes orbit should appear to be that of an overlapping horseshoe made of tightly wound kidney bean like spirals, with and orbital period of ~385 years. An Image of the orbit can be found on Wiegert's web page[16].

- Obtained result: The simulated orbit does indeed look like the one Wiegert suggest, but its orbital period is a hundred years shorter than predicted. It matches the 285 years seen in the results of the other experiments. The experiment was repeated three times, with initial conditions from Wiegert in units of AU, with the same initial conditions but converted to SI units and with initial conditions from Nasa's Horizon simulation program. The outcome was the same each time and looked like the graph below.



Graph No. 20

[16] Paul Wiegert's page:http://www.astro.uwo.ca/~wiegert/3753/3753.html

Hitting Earth with Cruithne, a bit of Science Fiction

- Setup: The simulation runs in the normal Helio Centric frame with a step size of one Earth day. Where each day has 10 RK4 steps. Cruithne's and Earth's positions are written to a .txt file. 27 days before Cruithnes closest approach during the first simulation year (which starts at the 1st April of 2001) a change in momentum should occur such that Cruithne will collide with Earth on the 20th December (the day of closest approach during the first year) of the simulation. The necessary momentum change was first estimated on the basis of a two body calculation, involving only the Sun and Cruithne. Initial position was Cruithnes position on the 23rd November and the final position was Earths position on the 20th December. The result of this estimation was used in the simulation and then by trail and error modified until a hit occurred.

- Obtained result: Cruithnes velocities at the 23rd November:

Cruithnes velocities:	old	new	change	change in %
v_x [m/sec]	-24232,24	-13429,17	10803,07	44,50%
v_y [m/sec]	9970,96	15480,86	5509,91	55,30%
v_z [m/sec]	4904,87	21756,28	16851,41	344%

To find out how big the momentum change is we first need to estimate Cruithnes mass. If we assume that its density if about the same as Earth's density[17] which is

$$\rho_{Earth} = \sim 5500 \, {}^{kg}\!/_{m^3}$$

and assume further that it is of spherical shape with a radius of 2,5 km[18] then we get:

$$M_{Cruithne} \approx \rho_{Earth} \times \frac{4\pi}{3} \times r_{Cruithne}^3 \approx 3{,}6 \times 10^{14} \, kg$$

Since the velocity changes Δv are of order 10^4 which means that the total momentum change Δp is of order:

$$\Delta p \approx \Delta v \times M_{Cruithne} \approx \sim 10^{18} \, {}^{m\,kg}\!/_{s}$$

To get a good idea of how much energy would be required to achieve this momentum change lets make a little *Gedankenexperiment*. If we assume that it would be possible to transfer energy into momentum without loosing any in the conversion process then we can estimate the required minimum energy as:

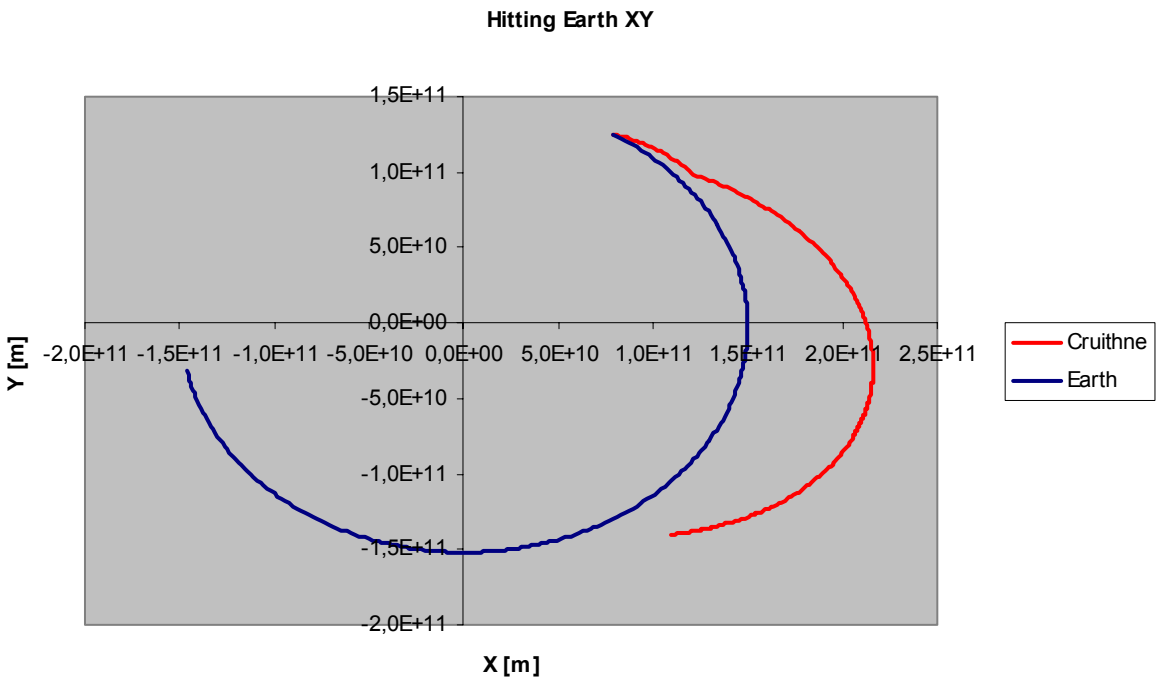
$$E \approx \frac{\Delta p^2}{2 M_{Cruithne}} \approx 10^{22} \, J$$

Since one ton of TNT relates to ~10^9J and the atomic bomb dropped onto Hiroshima was estimated to have the same energy as ~15000 tons of TNT which converts to ~10^{13}J. So the energy required to change Cruithnes path such that it would hit Earth would require at least 1.000.000.000 Hiroshima bombs.

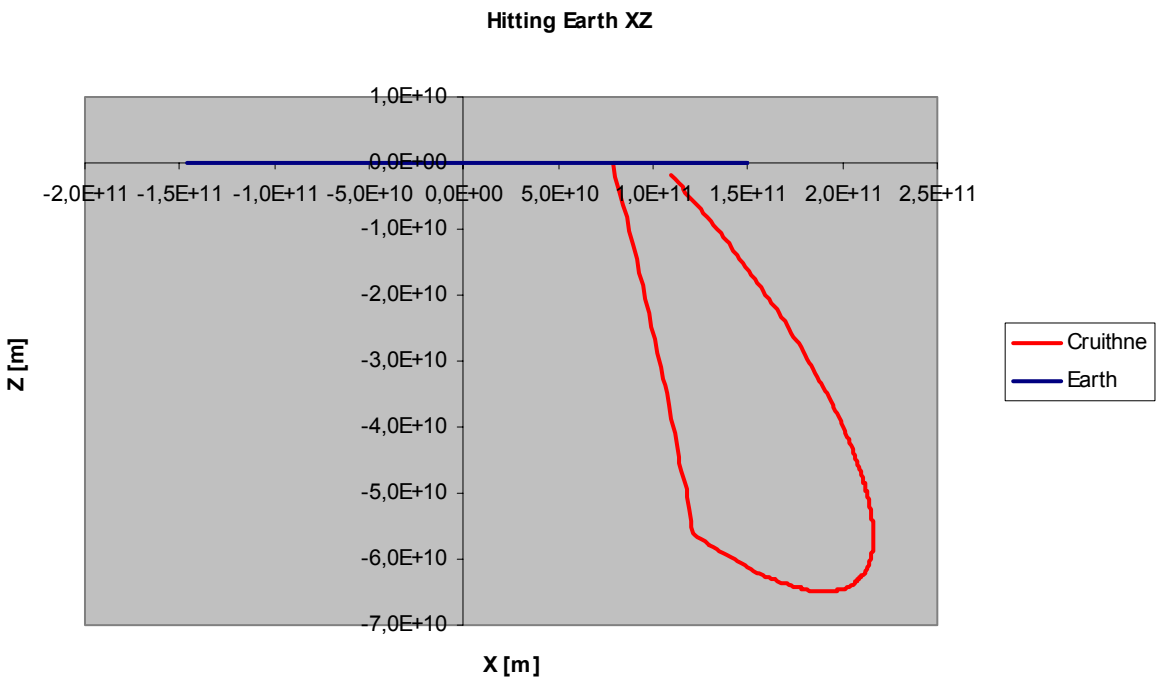
The change in Cruithnes orbit due to this momentum change can be seen in Graphs No. 21 and 22.

[17] Found information about Earth here: http://en.wikipedia.org/wiki/Earth

[18] Information from Wiegert 's homepage:http://www.astro.uwo.ca/~wiegert/3753/3753.html



Graph No. 21



Graph No. 22

Discussion

Although this simulation agrees qualitatively with the results that Wiegert *et al.* published in 1996, two points of disagreement remain.

The duration of one Earth year:

Paul Wiegert ran his simulation in a frame which used units of Au for distances, Au per year for velocities and the mass of the sun as a reference mass. In his frame the constant of Gravity has a value of $4\pi^2$ and a year is actually 1.000015 of an Earth year. This is because a unit year is defined as the time it takes for a body of unit mass to complete a 1 Au orbit around the sun, but since Earth is not mass less Kepler's Second Law gives a slightly shorter orbit time.
One possible explanation would be that something went wrong while converting the initial conditions in SI-units. However, the conversion was checked several times and no simple error (i.e. a typing mistake, etc.) was found. Running the code in Wiegert's units showed also the same disagreement. The next logical assumption would be that there is either a coding mistake or rounding errors due to numerical precision during the calculation. While rounding errors certainly do occur, the basic code testing described in the Method section of this report indicates that those errors are almost negligible. The error found in the 100 year test run with a satellite was of order 10^2m but since the orbital radius was 10^{11}m the total error range is just of order 10^{-9}. Now one might argue that this test involved only on body interacting with a fixed sun, but the test with two bodies orbiting there centre of mass gave a similar small error range, indicating that the code is working correctly. Since no explicit explanation was found, the value of a year was slightly adjusted in such a way that Earth would always come back to roughly the same position after each year (described in the "stroboscopic view" experiment).

The orbital time of the horseshoe orbit in the co-rotational frame:

Even more interesting is the fact that the simulated orbital time is exactly 100 years short of the predicted orbit time. Since the orbital time depends on the small variations between Earth's and Cruithne's orbital time, it seems independent of the small changes in the duration of a year. So a connection between the two mistakes seems unlikely. Also a wrong calculation in the code should influence both bodies i.e. Earth and Cruithne in the same way so their relative speeds and therefore the orbital time should be unaffected. Interesting is also the fact that the qualitative features like the kidney bean shaped orbit per year and the overlapping of the horseshoe occurs exactly as excepted, except that it takes 285 years instead of 385 years. Even Wiegert is puzzled about it and looked into his results again thinking that it might have been a typing error in his publication, unfortunately it is not. Since the simulation seems to work it appears that the missing 100 years might have to do with the initial conditions. So the test was repeated with Wiegert's initial conditions and units, then with Wiegert's units converted to SI units and initial conditions from NASA's Horizons simulation. The outcome was the same in all three cases. This basically leaves only one conclusion. There must be a very tricky mistake in the code, but even though some extensive testing and debugging was done it could not be located. The next debugging step would probably be to go through the code and explain it to somebody else with programming skills and hope that another person with a "fresh view" can spot the mistake.

Conclusion

The obtained results agree qualitatively with what Wiegert *et al.* found during there research in 1997, but disagree in the orbital periodic time of Cruithne's horseshoe orbit. Instead of the expected 385 years only 285 years where found. This mistake is insofar quite puzzling as all tests so far indicate that the simulation software is working correctly. Additional tests, like a change of the numerical integration algorithm or a step wise reduction of the number of planets in the simulation, could be used to narrow down the possible error source.
If the project was to be repeated, then more time should be spend validating the functionality of the used code before starting the experiments.

References

Wiegert *et al.*, An asteroidal companion to the Earth, Nature, vol.387, 685-686, 12 Jun 1997;

W.H.Press *et al.*, Numerical Recipes in C, 2nd Edition, Cambridge University Press, Feb 1993;

Project web page:
http://www.ph.ed.ac.uk/~p0125205/Solar/Solar.htm

Paul Wiegert's Cruithnes web page:
http://www.astro.uwo.ca/~wiegert/3753/3753.html

Numerical Recipes online at Cornell University:
http://www.library.cornell.edu/nr/cbookcpdf.html

Additional initial conditions from NASA's Horizon simulator. Can be accessed via *telnet*:
Address: ssd.jpl.nasa.gov *Port Number:* 6775

RK4 explained: http://www.library.cornell.edu/nr/bookcpdf/c16-1.pdf

Euler method: http://qucs.sourceforge.net/tech/node22.html

Information on Near Earth Objects:
http://neo.jpl.nasa.gov/
http://www.nearearthobjects.co.uk/

Information about the planets and asteroids:
http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

Historic information about the Picts: http://www.scotshistoryonline.co.uk/

Astronomical Resonance explained:
http://www.astro.uu.nl/~strous/AA/en//antwoorden/zwaartekracht.html - resonance
http://math.arizona.edu/~goriely/M254-2004/resonances.pdf

Namouni, Secular interactions of co-orbiting objects, 1999, Icarus, 137,293;

Information about Space-Time and Mercury:
http://theory.uwinnipeg.ca/mod_tech/node60.html

Biography of Nicolaus Copernicus: http://www.phy.hr/~dpaar/fizicari/xcopern.html

Information about Johannes Kepler: http://www.kepler.arc.nasa.gov/johannes.html

Mercury as example for precession:
http://phyun5.ucr.edu/~wudka/Physics7/Notes_www/node98.html

Further Internet Resources:
http://en.wikipedia.org/wiki/Main_Page
http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

Appendix

The printed appendix contains the source code and the java documentation of the applet version of simulation software and the e-mail correspondence with Paul Wiegert.

All experimental data, the source code for all non graphical simulation versions and the different initial conditions as well as the test code are on the CD-ROM which is handed in together with this report.

This was done due to fact that multiple software versions where used and the large amount of collected data. Printing all spread sheets containing the experimental results and all different software versions would be unreasonable.